# INVESTIGATION OF NATURAL CONVECTION IN CYLINDRICAL LIQUID LAYERS

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An experimental study has been made of heat transfer in cylindrical gaps under natural convection, using water and 96% ethanol. A comparison has been made of the convection coefficient in horizontal and vertical tubes, and the conditions for generating convection have been defined more accurately.

The transmission of heat by natural convection from a fine wire in a closed volume has not received much study. The investigation of this phenomenon is of interest for the elucidation of the convection mechanism of heat transfer.

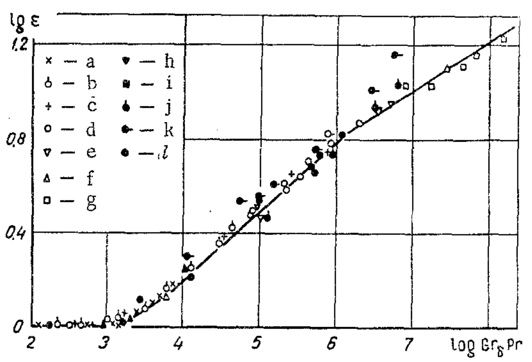


Fig. 1. The Kraussold-Mikheev curve $\varepsilon = f(\mathrm{Gr}_\delta \mathrm{Pr})$ in logarithmic coordinates for cylindrical layers with values of $\delta$ a) 7.5 mm; b) 17.5; c) 27.5; d) 42.5; e) 91.5; f) 191.5; g) 284.5 and h) for plane layers from the data of Nausselt, i) Schmidt, and Mull and Raier for j) vertical and k) horizontal layers, and *l*) Von Epp.

In measurements of molecular heat transport in liquids it is necessary to know the conditions for creating natural convection, and in a number of cases, to calculate a correction for convective heat transfer. Measurement of thermal conductivity is usually conducted in short tubes in very thin liquid layers, while basic knowledge of natural convection has been obtained in long and wide tubes. Laws of convective heat transfer determined experimentally in long and wide tubes are being applied to short and narrow tubes.

Convective heat transfer in liquids depends on the nature of the liquid motion. When heated, the liquid motion is determined by its physical parameters, the geometry of the space in which the heat transfer takes place, and on the orientation of the channels with respect to gravity forces.

In convective heat transfer investigations, wide use is made of the generalized correlations obtained by Kraussold and by Mikheev for the convective coefficient in the gap between coaxial tubes. These relations were obtained, however, on the basis of results of tests performed with quite a large gap between the cylinders. Recent test data on convection with small gaps have shown that the Kraussold curve cannot always be regarded as general. We therefore set up a special experiment with different gaps between cylinders, in the region not previously studied.

Using a similarity theory method, the solution of the system of differential equations describing heat transfer may be written in the form of functions which interrelate similarity criteria. When the system of differential equations cannot be integrated, the form of the functions is found experimentally. By generalizing the test data of various authors [1–4], Kraussold [5] obtained a relation for free-convection heat transfer in a bounded volume of liquid or gas. He took as the characteristic geometric dimension the thickness of the heat-transmitting layer, $\delta = (D_{out} - D_{in})/2$, for the coaxial cylinder case. His parametric equation was later improved by Mikheev [6], and the following relation obtained:

for the region $10^3 < \mathrm{Gr}_\delta \mathrm{Pr} < 10^6$,

$$\varepsilon = 0.105(\mathrm{Gr}_\delta \mathrm{Pr})^{0.3}; \tag{1}$$

for the region $10^6 < \mathrm{Gr}_\delta \mathrm{Pr} < 10^8$,

$$\varepsilon = 0.4(\mathrm{Gr}_\delta \mathrm{Pr})^{0.2}, \tag{2}$$

$$\varepsilon = \lambda_{ef}/\lambda, \tag{3}$$

$$\mathrm{Gr}_\delta = \beta g \delta^3 \Delta t/\nu^2, \quad \mathrm{Pr} = \nu/a, \quad a = \lambda/C_p \gamma. \tag{4}$$

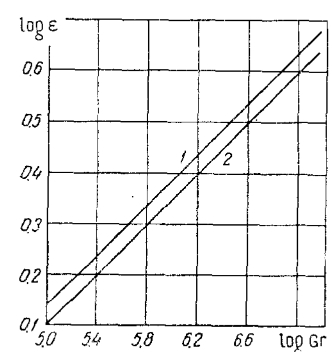


Fig. 2. Convection coefficients in 1) horizontal and 2) vertical cylindrical gas layers.

The test values of ε obtained by various authors as a function of the similarity criteria $\mathrm{Gr}_\delta \mathrm{Pr}$ determining convection lie on a single curve (Fig. 1). In the system of coordinates chosen, the magnitude of the convection coefficient does not depend on the shape and location of the heat transfer layer. Convection is observed to begin from values of the product $Gr_\delta Pr = 1000$. These results were obtained for gaps of from 7 to 285 mm.

In [7] convection coefficients obtained in vertical and horizontal cylinders are compared, other conditions being unchanged; the tests were made with two coaxial cylinders, with $D_{in} = 80$ mm, $D_{out} = 135$ mm, $\delta = 27.5$ mm, and the results are shown in Fig. 2, where it may be seen that the difference in convection coefficient for the vertical and horizontal cylindrical layers is small and has a systematic character.

Using a coaxial cylinder method ($D_{in} = 30$ mm, $\delta = 0.25$ mm), Selschopp [3] measured the thermal conductivity of liquid carbon dioxide. Others have measured the same quantity by the heated filament method: Kardos [8] used a platinum wire of diameter 0.1 mm in a tube of diameter 1.66 mm; Golubev [9], and Timrot and Oskolkova [10] made their measurements in tubes of diameter $\simeq 1$ mm with a $D_1 = 0.1$ mm platinum filament. The results of all these measurements differ considerably in some region of temperature and pressure, especially near the critical temperature. The scatter of the experimental data is mainly due to the fact that convection is easily generated in liquid carbon dioxide. When corrected according to the Kraussold-Mikheev equation the data do not converge to a single value.

In measurements of thermal conductivity of carbon dioxide in [11] by the heated filament method ($D_1 = 0.1$ mm, $D_2 = 1$ mm, $\delta = 0.45$ mm), a special investigation was made to determine convective heat transfer. The author found that convection began at a value of $Gr_\delta Pr = 2500$, but the $\varepsilon = f(Gr_\delta Pr)$ curve lay appreciably below the Kraussold-Mikheev curve. The Gr number was based on gap width $\delta$.

In the present paper an experimental investigation has been made of the dependence of convection coefficient in cylindrical gaps on the physical properties of the liquid, and on the size and location of the gap.

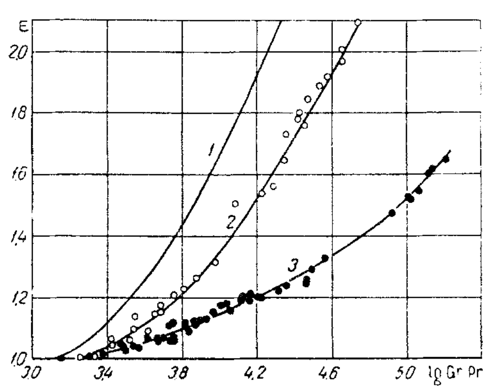


Fig. 3. Dependence of $\varepsilon$ on $Gr_\delta Pr$ (semilogarithmic scale): 1) Kraussold-Mikheev equation; 2 and 3) present results for horizontal and vertical cylinders, respectively; a) for water; b) for ethanol.

The essence of the method is an investigation of the effective conductivity in transmitting heat through a gap enclosed between two coaxial cylinders. The inner, heated cylinder is a platinum wire of diameter $D_1 = 0.1$ mm stretched out along the axis of a cylindrical glass tube, the inside wall surface of the glass tube being the cold cylinder. The gap sizes chosen lay in the range not previously studied: from 1.5 to 6.6 mm.

Dimensions of the Experimental Tubes, mm

| Filament diam. | Outer diam. of tube | Inner diam. of tube | Length of measuring section | Layer thickness (gap $\delta$) |
|---|---|---|---|---|
| 0.1 | 4.89 | 3.14 | 129 | 1.52 |
| 0.1 | 6.85 | 5.12 | 129 | 2.51 |
| 0.1 | 11.25 | 8.86 | 143 | 4.38 |
| 0.1 | 15.60 | 13.30 | 133 | 6.60 |

Similarity theory was used in generalizing the results, and during the work the dependence $\varepsilon = f(Gr_\delta Pr)$ for the conditions chosen was explored. The results were compared with the Kraussold-Mikheev correlation.

The thermal conductivity may be calculated from the Fourier equation as applied to cylindrical surfaces,

$$\lambda = \frac{\ln(D_2/D_1)}{2\pi l}\frac{Q}{\Delta t} = A\frac{Q}{\Delta t}. \quad (5)$$

The amount of heat transmitted Q is calculated from the value of the current $I_f$ and the voltage drop $U_f$ in the measuring section of the platinum filament

$$Q = I_f U_f. \quad (6)$$

The temperature drop is $\Delta t_1 = t_f - t_w$. The heated filament serves simultaneously as a resistance thermometer. A platinum wire mounted on the outer surface of the wall of the glass tube formed a second resistance thermometer. From the readings of these two, the temperatures of the heated filament and of the tube wall may be found. In calculating the effective thermal conductivity, a correction must be made for the temperature drop in the glass:

$$\Delta t_2 = \frac{\ln(D_3/D_2)}{2\pi l}\frac{Q}{\lambda_w}, \quad (7)$$

where

$$\lambda_w = 0.815(1 + 0.001 t_w) \ \text{W/m}\cdot{}^\circ\text{C}$$

(0.815 W/m · °C is the thermal conductivity of molybdenum glass at 0° C). It is known that the heat lost from the ends of the platinum heater in the case of a liquid is inappreciable, of the order of 0.5% [13].

The current and voltage in the resistance thermometers, and hence $t_f$, $t_w$, and Q, were measured by a compensation method, with the aid of a PPTV potentiometer.

The measurements were done in cylindrical molybdenum-glass tubes of various diameters, the dimensions being those in the table. During the measurements the tubes were located in a liquid thermostat with double glass walls, between which water at constant temperature was circulated.

In the present work the effective thermal conductivity of water and 96% ethanol was measured in tubes of different diameters, located horizontally and vertically.

For all the measured values of $\lambda_{ef}$, the convection coefficients $\varepsilon$ and the products $Gr_\delta Pr$ were calculated, using handbook values of the physical constants [12]; all the measured and calculated results are given in Fig. 3.

It may be seen from the results of our tests (Fig. 3) that convection with vertical and horizontal measuring tubes follows different curves, both of which lie considerably below the Kraussold-Mikheev curve. Therefore, the convection in the cylindrical gaps between the heated filament and the tubes with $\delta = 1.5$ to 6.5 mm is less than in large gaps with coaxial cylinders. Moreover, convection begins from $Gr_\delta Pr = 1700$.

Our experimental curve for vertical cylinders may be divided into two parts; for the region $2400 < Gr_\delta Pr < 35\,000$ the curve is described by the equation

$$\varepsilon = 0.465\,(Gr_\delta Pr)^{0.098}, \tag{8}$$

and for $35\,000 < Gr_\delta Pr < 170\,000$

$$\varepsilon = 0.28\,(Gr_\delta Pr)^{0.147}. \tag{9}$$

The divergence of the convection values in horizontal and vertical cylindrical layers increases with increase of the product $Gr_\delta Pr$.

SUMMARY

1. The results of the tests to investigate natural convection in heat transfer through a liquid layer contained between coaxial cylinders have been correlated in the form $\varepsilon = f(Gr_\delta Pr)$.
The experimental data fit two different curves; one describes convection in horizontal cylinders, the other in vertical cylinders. The course of both curves is independent of the nature of the liquid and of the layer thickness $\delta$.

2. The value of $\varepsilon$ in horizontal cylindrical layers is appreciably larger than in vertical layers.

3. According to our data, convection in both vertical and horizontal cylindrical layers is considerably lower than given by the Kraussold-Mikheev equation.

4. Convection in cylindrical layers for gap width 1.5 to 6.5 mm appears at $Gr_\delta Pr = 1700$.

NOTATION

$\delta$—thickness of layer; $\varepsilon$—convection coefficient; $\lambda_{ef}$—effective thermal conductivity; $\lambda$—thermal conductivity; $Gr_\delta$—Grashof number; $\beta$—coefficient of volume expansion; g—acceleration due to gravity; $\Delta t_1$—temperature difference; $\nu$—kinematic viscosity; Pr—Prandtl number; a—thermal diffusivity; $C_p$—specific heat at constant pressure; $\gamma$—specific weight; $l$—length of measuring section; $D_2$—internal diameter of glass tube; $D_1$—diameter of heater wire; Q—heat release; $t_f$—temperature of filament; $t_w$—temperature of internal wall surface; $D_3$—external diameter of glass tube; $\lambda_g$—thermal conductivity of glass at temperature t; $I_f$—heater current; $v_f$—voltage drop in measuring section of heated filament; $\Delta t_2$—temperature drop in glass; $D_{in}$—outer diameter of inner cylinder; $D_{out}$—internal diameter of outer cylinder.

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